

AN EMPIRICAL INVESTIGATION OF A DYNAMIC BRAKE LIGHT CONCEPT FOR REDUCTION OF REAR-END COLLISION ACCIDENTS DURING EMERGENCY BRAKING

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A potential application of optical looming manipulation is investigated: a dynamic brake light concept for potentially reducing the frequency of rear-end collisions in automobile driving. In a low-fidelity driving simulator, forty participants were instructed to follow a leading vehicle (LV) and appropriately respond to braking of the LV, under three visibility conditions: day, night-time with headlights, and night-time without headlights. During some LV braking events, separation and size of the brake lights of the LV were imperceptibly expanded or contracted, at one of two levels, to simulate an *effective virtual time shift* in the headway of the LV. Results show that this concept was effective only for very poor visibility conditions: at night with no headlights, for which LV brake lights were most salient. Subjects generally braked sooner when confronting a LV with expanding brake lights, and later for contracting lights, in comparison with the no manipulation case.

INTRODUCTION

Although current brake light systems indicate whenever the driver of a leading vehicle (LV) has a foot on the brake, this information is frequently insufficient for helping the following driver (FD) in properly judging *when* to brake, *how hard* to brake, or *whether* it is necessary to brake at all. While in most circumstances normally available visual information may be quite adequate for such judgments, in more extreme situations, such as nighttime and poor visibility conditions, there is always a danger that the FD will not be able to effect an adequate tradeoff between braking too abruptly (and thereby risking a skid and/or rear-end collision from behind), and braking too slowly (with the consequent danger of collision with the car ahead). Much consideration has been given to whether, and how, the intensity of a LV braking maneuver can be indicated to the traffic behind, with the primary focus of most studies to date being on reducing the frequency of rear-end collisions by allowing hazardous situations to be detected with sufficient time to permit appropriate braking profile control.

Many ideas have been proposed for informing the FD about braking emergencies. For example, based on an examination of various approaches to display brake force, including flashing lights, bar graph displays, and changing brake light size, brightness, colour, contour, location, and number of brake lights, Gail et al (2001) suggested a *two-stage brake force display* as a practical and feasible optimized rear signal pattern, for which the area and luminance of brake lights would change from a normally smaller, lower luminance to a larger, higher luminance state for vehicle deceleration rates greater than 7m/s^2 . Another proposal is an additional *imperative brake light signal* (Tang, 1989), which would light up only when the emergency brake is activated and would be useful in informing the FD that he should start to brake quickly and with maximal deceleration. These examples share the common concept of communicating an emergent situation to the FD by making use of “emergency” braking signals.

One potential disadvantage of such systems, however, is that it would not be unexpected for a significant number of following drivers (FDs) to adjust their subjective acceptable risk thresholds (Wilde, 1982) to accommodate to what they perceive to be the operational properties of such emergency braking systems, after having encountered them in action a number of times during driving. With that in mind, we propose that, to overcome that particular shortcoming, an ideal system would be one for which the following driver (FD) would be *unaware* that he is being warned to brake more rapidly but would nevertheless be compelled do so – involuntarily.

Research has shown that drivers are able to control braking based on information perceived from *optical looming*, that is, expansion of the retinal images of approaching objects (Lee, 1976; van der Horst, 1990, 1991; van Winsum & Heino, 1996). In our own simulated driving experiments, we have found a direct relationship between optical looming and control of braking (Li & Milgram, 2004a; 2005). In those experiments FDs were found to advance or delay their braking behaviour in a predictable fashion while following a lead vehicle (LV) whose optical image was manipulated according to either an *effective virtual displacement (EVD)* in distance or an *effective virtual time shift (EVTS)*. The hypothesis supported there is that, when the FD brakes sooner in response to an expanding LV, or later to a contracting LV, this behaviour is a result of manipulation of the optical looming cue. (Similar patterns have also been observed for other psychomotor tasks, including ball catching, bicycle braking, and even animal running tasks. (Savelsbergh et al., 1991, 1993; Wann & Rushton, 1995a; Sun, Carey & Goodale, 1992; Sun & Frost, 1998))

As a potential application of Li & Milgram's findings, a dynamic automobile brake light system has been suggested, as a possible means of reducing the rate of occurrence of rear-end collisions (Li & Milgram, 2004b). According to that proposal, whenever the leading driver brakes rapidly and the vehicle detects a following vehicle (FV) within a certain specified

distance (assuming the presence of gap estimation technology), both the *separation* and *size* of the brake lights of the LV will automatically expand, *continuously*, to amplify the natural optical looming which occurs during braking. Based on earlier research showing that optical looming of the visual image formed by the LV brake lights is an important source of information for the FD to regulate speed and inter-vehicle distance (Janssen et al, 1976; Liebermann et al, 1995), the argument behind the present research is that, by manipulating that cue as needed, it should be possible to shorten the FD's perception of time-to-collision (TTC) with the LV and cause him to brake sooner, to provide additional time (along the order of 100's of ms) for effecting appropriate control actions.

The objective of the experiment reported here was to empirically investigate whether and how such a dynamic brake light system might influence drivers' braking behaviour, in a low-fidelity driving simulator. Specifically, we have investigated whether manipulating the separation and size of the brake lights of the LV while it is decelerating can make FDs modify their braking behaviour accordingly. In addition, to ascertain whether such a phenomenon, if observed, might be a consequence simply of the *presence* of our manipulation, rather than of its prescribed pattern, our experiment comprises manipulations in both directions, that is, both expansion and contraction (even though any operational system would logically comprise only expansion), and at two levels for each. Our hypotheses are thus (i) that significant effects on FD braking behaviour can be expected through manipulation of only brake lights, (ii) that LV brake light expansion will cause advanced braking while contraction will retard braking, and (iii) that modifying the magnitude of the manipulation will cause a concomitant change in resulting braking behaviour. A fourth aspect of our investigation involved a comparison of daytime versus night-time driving, under different illumination conditions, with the hypothesis being (iv) that manipulation of the brake light optical looming properties will have its greatest effects for the lowest visibility conditions, for which the brake light cue is most relevant for assisting in FV braking.

METHOD

Participants

Forty paid male volunteers participated in the experiment. They were 18-36 years old (mean=24; SD=5.1) with normal or corrected-to-normal vision and naïve to the purpose of the experiment. All had full driving licenses, with 3-15 years of driving experience (mean=6; SD=3.3). Our reason for limiting the subject population to young male drivers was to give more power to our experiment, by allowing us to focus on the optical cue manipulation factor. In doing so, any existing significant differences in braking behaviour between manipulation and no manipulation conditions could be more easily identified, without other variances caused by gender or age.

Apparatus

Experiments were conducted using a low-fidelity driving simulator, running on a normal PC. The roadway scene was projected onto a large screen, using a LCD projector. The

steering wheel and gas/brake pedal were standard game control devices. Figure 1 shows three sample scenes for the three scenarios corresponding to the three visibility conditions for this experiment (from left to right): *day time*, *night-time with FV headlights*, and *night-time without FV headlights*.

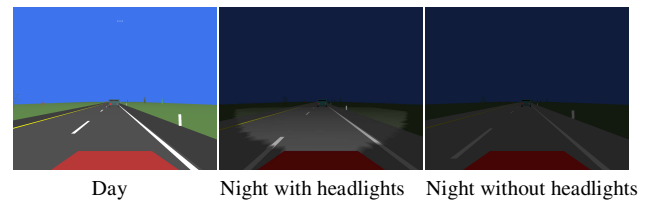


Figure 1. Experimental scenes for the three following scenarios. (The shape at the bottom is the hood of the following vehicle.)

Task

Subjects were instructed to "safely follow" (without overtaking) a LV at a required following distance (33m) on a simulated straight highway and to appropriately respond to braking of the LV, just as they would "under real driving conditions". Although a certain tolerance ($\pm 10\text{m}$) in following distance was allowed, voice messages were given to help subjects stay within the required following distance range. Whenever the following vehicle (FV) fell too far behind the LV, a message of "Too far!" was given; conversely, whenever the gap became too small, they were told "Too close!" Because braking behaviour depends on following distance and relative speed between vehicles, we kept these parameters approximately constant across braking events. In particular, with the speed of the LV at 80 km/h, a braking event was triggered only when the speed of the FV was within the range 77-83 km/h and whenever the following distance was within $\pm 1\text{m}$ of the required following distance.

To motivate subjects to attend to the task and brake according to the instructions, a point system was implemented, based on some performance indices, including excessive braking, number of rear-end collisions, and frequencies of 'too far' and 'too close' warnings. Based on the average points accumulated over trials, a monetary bonus was awarded.

Procedure

The study was conducted in one session. Participants first filled out a demographic questionnaire and received both written and oral instructions describing the experimental platform and the task. This was followed by 3 training trials, one for each of the 3 scenarios, and 9 real trials, 3 for each of the 3 scenarios. At the end of the experiment, participants filled out a questionnaire which probed the strategies used for following and braking. Feedback about the experiment and the simulator was also solicited.

At the beginning of each trial, a vehicle would pass the subject's vehicle in the left lane, move to the right lane ahead, and then become the lead vehicle (LV). During each trial, the LV randomly braked 20 times, at one of 4 different deceleration rates, accompanied by one of 5 different ways of manipulating the separation and size of the brake lights of the LV. The deceleration rates were divided into one 'low rate': $\{4 \text{ m/s}^2\}$ and 3 'high rates': $\{6, 8, 10 \text{ m/s}^2\}$. During high deceleration, the LV slowed to a complete stop, but during low deceleration, the LV

slowed to only 60 km/h. In both cases the LV commenced reaccelerating to 80 km/h 2 seconds later. The low deceleration rate thus provided a set of distracting events, to prevent subjects from habituating to hard braking.

Each trial lasted about 10 minutes, with the whole experiment lasting about 2.5 hours, including breaks between trials. Manipulation conditions and deceleration rates were randomised for every trial, and the order of presentation of trials was counterbalanced across participants.

Manipulation of the Optical Looming Cue through Size and Separation of the Brake Lights during Braking

During some of the braking events, the size of the triangle formed by the two bottom brake lights and the one high centre brake light of the LV was manipulated continuously, in one of two directions: expansion or contraction, and at one of two levels: 0.2s and 0.4s of *effective virtual time shift (EVTS)*, as illustrated in Figure 2 and explained below.

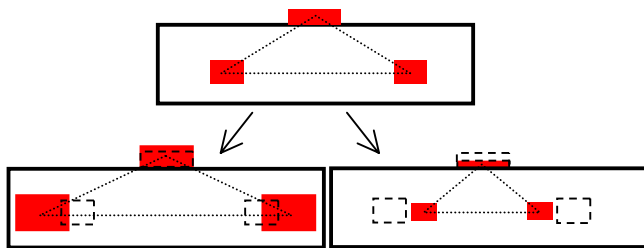


Figure 2. Sketch showing manipulation of separation and size of the brake lights during LV braking. Top represents normal pattern of brake lights. Bottom left represents expanded brake lights and bottom right represents contracted brake lights, relative to normal pattern (dashed lines).

The magnitude of the size change of the LV *brake light triangle (BLT)* image was designed to mimic the expansion of the retinal images of the same triangle, of constant size, as if the array of lights were located behind (while expanding) or in front of (while contracting) their actual position – i.e. closer to or further away from the FV. Note that *only the rate of change of size (optical looming) of the BLT, not its actual position, was manipulated*. To provide an advanced or retarded EVTS for each frame during braking, the software recomputed the size of the BLT, which was then scaled in accordance with the EVTS of the BLT. The algorithm was that $R(t)$, the factor scaling the rendered array of brake lights, was changed on each frame, according to (Wann & Rushton, 1995b; Li & Milgram, 2004b):

$$R(t) = [Z(t) / (Z(t) - RV(t)*\Delta T)]$$

where ΔT is the required time manipulation (EVTS); $Z(t)$ is the instantaneous distance between the subject’s viewpoint and the rear of the LV; $RV(t)$ is the relative velocity between FV and LV. $R(t)$ was changed only when $RV(t)>0$, that is, whenever the FV was closing in on the LV. Note that $D(t)=RV(t)*\Delta T$ is the *effective offset distance*, or (continuously changing) *virtual distance* corresponding to the BLT’s optical expansion. (Reiterating, the BLT was not really displaced by the distance D ; only its rate of change of size was changed, resulting in a virtual time shift.)

In designing the experiment ΔT had to be chosen not too small, so as to increase the probability of observing an effect, but not

too large, so as to ensure that the manipulation remained imperceptible. Taking into account the possible range of RV values, given the constraints of this experiment, five levels of manipulation (ΔT) were used: 0s, ± 0.2 s, ± 0.4 s. In other words, negative values corresponded to retarded optical looming cues (LV effectively farther away), positive values to advanced displacement (LV effectively closer), and 0 to no manipulation. Our reasoning in this design was that, if expanding the BLT during braking were to reduce perception of TTC, the subjects should generally brake sooner on expansion events ($+0.2$ and $+0.4$ s), and the opposite should occur for contraction events (-0.2 and -0.4 s), relative to no manipulation events (0s). In addition, the magnitude of any effects should correspond to the magnitude of manipulation, with proportionately larger effects for 0.4s relative to 0.2s manipulations.

In addition to the above, careful attention was paid to potential interactions between the brake lights and the rear of the LV. In order to reduce conspicuous distortions of the manipulated BLT array relative to the rear of the LV, the largest (for expansion) and smallest (for contraction) values of scaling factor $R(t)$ were set to 1.333 and 0.677 respectively.

Furthermore, as stated earlier, it was reasoned that the effect of manipulations should be more pronounced whenever the brake lights are comparatively more visible, and thus more important. Consequently, three illumination conditions were compared: daytime with FV headlights, night-time with FV headlights, and night time without FV headlights, with any effects of the optical looming manipulations expected to increase in that order.

Data Analysis

The following indices were measured and recorded during each braking event (with 40ms resolution): Time of first Pressing Brake pedal (TPB); Time of Maximum Braking force (TMB); Maximum Braking Force (MBF); Minimum Time-to-Collision (MTTC). For all time related indices, time was computed from the instant of LV braking.

RESULTS

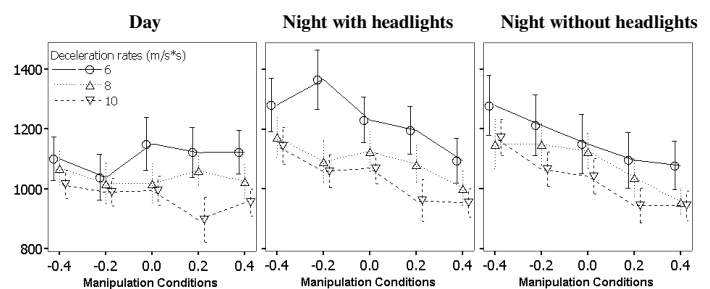


Figure 3. Time of first pressing brake (TPB) pedal (in ms). (In all graphs, results are across 3 scenarios and 3 deceleration rates, under 5 manipulation conditions. Error bars represent 95% confidence intervals. Positive, zero and negative times represent after, at, and before the start of LV braking.)

Time of First Pressing Brake Pedal (TPB)

As shown in Figure 3, expanding ($+0.2$, $+0.4$ s) or contracting (-0.2 , -0.4 s) the separation and size of the brake lights of the LV during braking did affect braking in conjunction with the

manipulation levels during night-time but not day conditions. For day conditions alone, a two-way within-subjects ANOVA indicated a significant effect of deceleration rate ($F(2,118)=118, p<0.001$) but no effect of manipulation (ΔT) ($F(4,116)=1.9, p=0.114$), and no significant interaction ($F(8,112)=2.5, p=0.017$). For night-time with headlights, a similar analysis revealed significant main effects of both manipulation (ΔT) ($F(4,116)=21.5, p<0.001$) and deceleration rate ($F(2,118)=38.8, p<0.001$), with no interaction effect ($F(8,112)=1.5, p=0.149$). For night-time without headlights, a similar ANOVA again indicated significant main effects of manipulation (ΔT) ($F(4,116)=13.8, p<0.001$) and deceleration rate ($F(2,118)=14.7, p<0.001$), with no interaction ($F(8,112)=0.9, p=0.532$).

Time of Maximum Braking Force (TMB) and Maximum Braking Force (MBF)

As illustrated in Figure 4, for the daytime scenario two two-way within-subject ANOVAs showed that the ΔT manipulation did not produce any significant differences for either time of maximum braking force (TMB) ($F(4,116)=2, p=0.101$) or maximum braking force (MBF) ($F(4,116)=1, p=0.407$), although a significant effect of deceleration rate was found for both (TMB: $F(2,118)=27.6, p<0.001$; MBF: $F(2,118)=211.4, p<0.001$). Neither variable had any significant interactions (TMB: $F(8,112)=1, p=0.412$; MBF: $F(8,112)=1.9, p=0.071$).

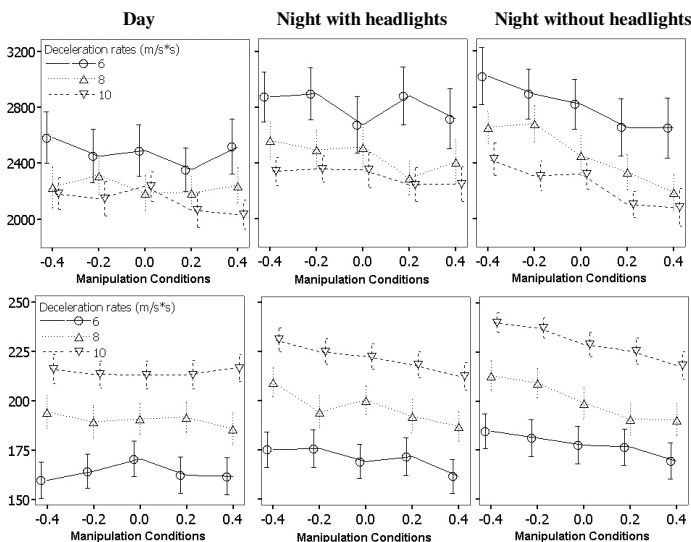


Figure 4. Top: Time of maximum braking force, TMB (in ms) and bottom: maximum braking force, MBF (range is between 0-255).

For the night-time with headlights scenario, for time of maximum braking force, a two-way within-subject ANOVA showed a significant effect of deceleration rate ($F(2,118)=45.9, p<0.001$), but no effect of ΔT manipulation ($F(4,116)=2.3, p=0.062$), and no interaction ($F(8,112)=1.2, p=0.323$). However, for maximum braking force (MBF), both a significant ΔT manipulation main effect ($F(4,116)=14.9, p<0.001$) and a significant deceleration effect ($F(2,118)=251, p<0.001$) were found, without any significant interactions between them ($F(8,112)=1.5, p=0.171$). This suggests that, during simulated driving at night with headlights, although the expansion manipulation did not cause subjects brake to maximum braking

force earlier, it did reduce the maximum braking force (i.e. less emergency braking), due to earlier commencement of braking.

For the scenario of night driving without headlights, the general trend is very clear in Figure 4: subjects used a smaller maximum force when pressing the brake pedal earlier on expansion events (0.2s, 0.4s) and a larger maximum force later on contraction events (-0.2s, -0.4s) relative to no manipulation events (0s). This suggests that braking earlier for the expansion events left a bigger margin of safety in terms of maximum braking force (i.e. less emergency braking). Alternatively, later braking had to be compensated for by harder braking, which is potentially more dangerous. For TMB, a two-way within-subject ANOVA showed both a significant ΔT manipulation effect ($F(4,116)=17.9, p<0.001$) and a significant deceleration rate effect ($F(2,118)=39, p<0.001$), with no interaction ($F(8,112)=1.1, p=0.366$). For MBF, ANOVA again showed a significant ΔT manipulation effect ($F(4,116)=21.1, p<0.001$) and a significant deceleration rate effect ($F(2,118)=196.7, p<0.001$), with no interaction ($F(8,112)=1.1, p=0.355$).

Minimum Time-to-collision (TTC)

Mean minimum time-to-collision estimates during braking are an indication of the potential imminence of a collision, and as such are potentially very valuable for directly reflecting any influence of effective virtual time shift (ΔT) manipulation. As seen in Figure 5, subjects tended to achieve longer minimum TTC values during braking on expansion events (0.2s, 0.4s) and shorter TTC values on contraction events (-0.2s, -0.4s) relative to no manipulation events (0s) only for day time driving for 10m/s² deceleration and night driving for deceleration rates of 8 and 10m/s². This may be because the same imposed ΔT shift has less influence on the larger actual TTC between FV and LV produced by the slower deceleration rates during braking.

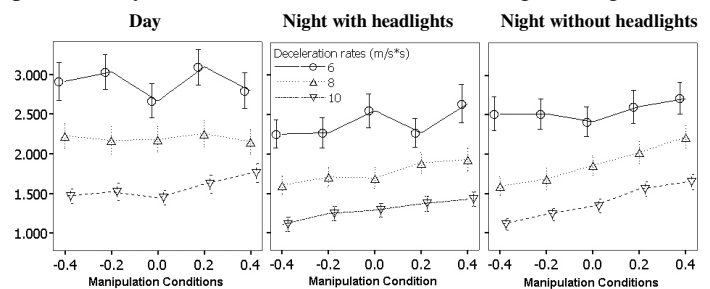


Figure 5. Minimum Time-to-collision (TTC) (in seconds)

For daytime, a two-way within-subject ANOVA showed a significant ΔT manipulation effect ($F(4,116)=6.1, p<0.001$) and a significant effect of deceleration rate ($F(2,118)=215.2, p<0.001$), but with a significant interaction ($F(8,112)=3.6, p=0.001$). Three separate one-way within-subject ANOVAs were then conducted, for each of the three deceleration rates, showing significant ΔT manipulation effects only for a deceleration rate of 10m/s² ($F(4,116)=5.653, p<0.001$), but not for 6 or 8 m/s² (6m/s²: $F(4,116)=2.3, p=0.055$; 8m/s²: $F(4,116)=0.3, p=0.895$). (From Figure 5, we see that this effect is significant only for the expansion direction).

For night driving with headlights, a two-way within-subject ANOVA showed a significant ΔT manipulation effect

($F(4,116)=15.3$, $p<0.001$) and a significant deceleration rate effect ($F(2,118)=178.5$, $p<0.001$), with no interaction ($F(8,112)=2.4$, $p=0.021$). Three one-way ANOVAs were then conducted separately for the three deceleration rates, showing a significant ΔT manipulation effect for deceleration rates of 8 and 10m/s^2 (8m/s^2 : $F(4,116)=4.2$, $p=0.002$; 10m/s^2 : $F(4,116)=7$, $p<0.001$), but not for 6m/s^2 (6m/s^2 : $F(4,116)=3.1$, $p=.014$).

For night driving without headlights, a two-way ANOVA showed both a ΔT significant manipulation effect ($F(4,116)=25.9$, $p<0.001$) and a significant effect of deceleration rate ($F(2,118)=165.9$, $p<0.001$), with a significant interaction ($F(8,112)=2.7$, $p=0.009$). Three one-way within-subject ANOVAs were then conducted for each of the deceleration rates separately, showing significant ΔT manipulation effects for deceleration rates of 8 and 10m/s^2 (8m/s^2 : $F(4,116)=14$, $p<0.001$; 10m/s^2 : $F(4,116)=26.6$, $p<0.001$), but not for 6m/s^2 (6m/s^2 : $F(4,116)=1.2$, $p=.316$).

Questionnaire

At the end of experiment, subjects were asked, "In terms of the simulation of the automobile driving task, and within the obvious constraints of our driving simulator, did the lead vehicle image appear normal during the braking portion of the experiment?". All 40 subjects said it appeared normal to them. They were then further asked, "Actually, in this experiment we manipulated (by expanding or contracting) the separation and size of the image of the three brake lights on the lead vehicle during braking, but only for some of the braking events. Were you aware of this manipulation before reading this question?". 25 of them (62.5%) said "Not at all". 13 of them (32.5%) said "No, but I noticed something abnormal," with comments like "When getting very close to the LV, sometimes I saw the brake lights moving apart or closer, but I thought it was a software glitch or was a normal way to simulate braking. I didn't pay much attention to it". Only 2 (5%) of the 40 participants answered "Yes" with similar comments, but said that this didn't change their braking behaviour, i.e. they didn't adopt any particular strategy to compensate for the unreliable cue available from the brake light image.

DISCUSSION

In summary, the concept of a dynamic brake light system shows some promise for causing drivers to brake sooner in emergencies during visibility conditions such as night-time driving along poorly illuminated roads, where drivers' ability to detect slowing traffic ahead tends to be diminished the most. In particular, our experiment showed that commencement of braking was 50-150 ms faster, and minimum TTC values during braking were up to 50-200 ms longer. Compared to our earlier experiment in which the whole size of the LV was manipulated (Li & Milgram, 2005) – a much more obvious cue than only brake lights – the effect of ΔT manipulation in this experiment is much smaller. Specifically, the actual minimum TTC shift observed in this experiment was about 0.1s, whereas it was about 0.3s in the earlier experiment, for $\Delta T=0.2\text{s}$ in both. Nevertheless, it is well known that reductions in brake reaction

time of as little as 100ms (or 2.22m at 80km/h), as achieved in this experiment, can contribute in principle to a significant reduction in stopping distances during emergency braking in real life automobile driving, and could thus conceivably lead to significant reductions in both the frequency and severity of rear-end collisions (Sivak & Flannagan, 1993). Our results also support the theory that optical looming plays an important role in automobile driving, and that this manifests itself in terms of drivers using estimates of TTC information for braking control.

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